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Magnetic Conductivity and Chern-Simons Term in Holographic Hydrodynamics of Charged AdS Black Hole

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Abstract

We study the effects of the Chern-Simons term in the hydrodynamics of the five-dimensional Reissner-Nordström-AdS background. We work out the decoupling problem of the equations of motion and calculate the retarded Green functions explicitly. We then find that the Chern-Simons term induces the magnetic conductivity caused by the anomaly effect. It is increasing function of temperature running from a non-zero value at zero temperature to the twice the value at infinite temperature.

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1 Introduction

Much efforts to use AdS/CFT correspondence [1–3] to understand the strongly coupled field theory have been performed after the discovery of low viscosity in the gravity dual [4]. The way to include a chemical potential was figured out in the context of probe brane embedding [5–12]. Even an attempt to map the entire process of RHIC experiment in terms of the gravity dual [13] was made. In spite of differences in QCD and the supersymmetric gauge theories, it is expected that some of the properties are shared based on the universality of low energy physics. In this respect, the hydrodynamics is particularly interesting. The calculation scheme for transport coefficients is to use Kubo formula, which gives a relation to the low energy limit of Wightman Green functions. In AdS/CFT correspondence, one calculates the retarded Green function which is related to the Wightman function by fluctuation-dissipation theorem. Such scheme has been developed in a series of papers [14–18].

In our previous papers [19–22], we used the five-dimensional Reissner-Nordström-Anti-deSitter background (RN-AdS$_5$) [23–27] to model the dense baryonic media. However, here the $U(1)$ charge in RN-AdS$_5$ black hole background is the R-charge and the Chern-Simon term describes the anomaly effect. RN-AdS$_5$ can be also obtained from the STU solution [28] by taking its diagonal-part, and the Chern-Simons term is essential since the $U(1)$ is anomalous unlike the baryon $U(1)$ charge case.

It is generally known that the Chern-Simons term has relations with the parity symmetry breaking and the triangle anomaly, and its importance was pointed out in [29]. It was shown that DC conductivities appear as coefficients concerning pseudo-vectors and pseudo-tensors in the derivative expansion of energy momentum tensor and conserved current, when the background is boosted and the charge and mass vary slowly with the spacetime coordinates [30–32]. As for the Chern-Simons term in the RN-AdS$_5$ background theory, new understanding has been obtained recently [33]. In their study, from the fact that the Chern-Simons term has to do with the triangle anomaly, a relation between its coefficient and the vorticity-induced term has been newly revealed. Studies with the Chern-Simons term to obtain a description for the real QCD plasma in RHIC have been performed in [34, 35]. In [34], the time-dependent chiral magnetic conductivity has been calculated, in which the property of the five-dimensional Chern-Simons term corresponding to the

four-dimensional anomaly plays an important role. On the other hand, in [35], exploiting multiple $U(1)^3$ symmetries in the STU model, the viscous hydrodynamics of hot conformal field theory plasma with $U(1)^{N_f}$ flavor symmetry as well as $SU(2)_{\mathrm{I}}$ non-Abelian iso-spin symmetry has been studied.

The aim of this paper is to complete the leading order hydrodynamics in the equations of motion for RN-AdS$_5$ with the Chern-Simons term. Notice that the presence of the Chern-Simons term does not change the background spacetime, but it can change the perturbative spectrum around the solution. We calculate the retarded two-point Green functions explicitly and get the transport coefficients in the presence of Chern-Simons term.

This paper is organized as follows: In section 2, we describe the regularized action with the Chern-Simons term and the RN-AdS$_5$ background obtained from the STU model in the equal three charges. The perturbations from the background are solved under the hydrodynamic approximation in section 3. In this procedure, we perform the decoupling procedure by using the master variables. In section 4, we obtain the two-point retarded Green functions in the dual Quark-Gluon-Plasma (QGP) via AdS/CFT correspondence. We then investigate the effect of the Chern-Simons term in the bulk gravity to the hydrodynamics of the dual QGP, and summarize this study in section 5. In Appendix A, we give the procedure to calculate the retarded Green function of the boundary theory in AdS/CFT correspondence formulated by Son and Starinets.

2 Setup in the dual gravity

We first consider the STU solution [28]. In the AdS/CFT correspondence, this solution can be understood as a near horizon geometry of D3-branes and three $U(1)$ charges are R-charges which come from the Kaluza-Klein momenta in S^5. Here, we consider the solution with all the three charges are equal. We introduce perturbations around this solution, and calculate the Green functions in the dual CFT by using the correspondence. For simplicity, we impose a constraint such that the three $U(1)$ gauge fields take the same value. Then, the action of the STU model reduces to the following:

$$S = \frac{1}{16\pi G_5}\int \mathrm{d}^5x\sqrt{-g}\left(R+\frac{12}{l^2}\right) + S_{\mathrm{Maxwell}} + S_{\mathrm{CS}}\ , \tag{2.1}$$

where the parameters l is the radius of AdS space. $S_{\rm Maxwell}$ is the action for $U(1)$ gauge field,

$$S_{\rm Maxwell} = -\frac{1}{4e^2}\int {\rm d}^5x\sqrt{-g}\mathcal{F}_{mn}\mathcal{F}^{mn} \ , \tag{2.2}$$

with $\mathcal{F}_{mn} = \partial_m\mathcal{A}_n - \partial_n\mathcal{A}_m$, while $S_{\rm CS}$ is the Chern-Simons term,

$$S_{\rm CS} = \frac{\kappa}{3}\int {\rm d}^5x \ \varepsilon^{lmnpq}\mathcal{A}_l\mathcal{F}_{mn}\mathcal{F}_{pq} \ , \tag{2.3}$$

where κ is the parameter, through which the effect of the Chern-Simons term appears.

Almost same action without the Chern-Simons term can be obtained from the bulk-filling D7-brane in the AdS background [22]. In the case of the bulk-filling D7-brane, the $U(1)$ charge can be identified with the overall $U(1)$ part of the $U(N_f)$ flavor symmetry, and the gauge coupling constant e and the gravitational constant G_5 can be written as

$$\frac{l}{e^2} = \frac{N_cN_f}{(2\pi)^2} \qquad \text{and} \qquad \frac{l^3}{G_5} = \frac{2N_c^2}{\pi} \ , \tag{2.4}$$

where N_c and N_f denote numbers of D3- and D7-brane, respectively. Even though the STU solution is not correspond to the SQCD which comes from the D3-D7 system, it can be expected that the dual field theory shares some properties with the SQCD.

The equations of motion are given as

$$R_{mn} - \frac{1}{2}g_{mn}R - \frac{6}{l^2}g_{mn} = 8\pi G_5T_{mn} \ , \tag{2.5a}$$

$$-\frac{1}{e^2}\nabla_n\mathcal{F}^{mn} + \frac{\kappa}{\sqrt{-g}}\varepsilon^{mlnpq}\mathcal{F}_{ln}\mathcal{F}_{pq} = 0 \ , \tag{2.5b}$$

where T_{mn} is the energy-momentum tensor,

$$T_{mn} = \frac{1}{e^2}\Big(\mathcal{F}_{mk}\mathcal{F}_{nl}g^{kl} - \frac{1}{4}g_{mn}\mathcal{F}_{kl}\mathcal{F}^{kl}\Big) \ . \tag{2.6}$$

Since the presence of the Chern-Simons term does not change the background spacetime from that in the bulk-filling D7-brane model, it can be seen that the following RN-AdS$_5$ background can be a classical solution,

$$\begin{aligned} {\rm d}s^2 \ &= \frac{r^2}{l^2}\Big(- f(r)({\rm d}t)^2 + ({\rm d}\vec{x})^2\Big) + \frac{l^2}{r^2f(r)}({\rm d}r)^2 \ . \\ \mathcal{A}_t \ &= -\frac{Q}{r^2} + \mu \ , \end{aligned} \tag{2.7}$$

with

$$f(r) = 1 - \frac{ml^2}{r^4} + \frac{q^2l^2}{r^6} = \frac{1}{r^6}(r^2 - r_0^2)(r^2 - r_+^2)(r^2 - r_-^2) \ ,$$

and q is related to Q by

$$q = 4\sqrt{\frac{\pi G_5}{3e^2}}\, Q \ . \tag{2.8}$$

Q denotes the $U(1)$ charge induced from the bulk-filling D7-brane. The parameters m and q correspond to the mass and charge of the AdS space, respectively. The explicit forms of $r_0(= -r_+^2 - r_-^2)$ and $r_\pm$ are given by

$$\begin{aligned} r_0^2 &= \left(\frac{m}{3q^2}\left(1 + 2\cos\left(\frac{\theta}{3} + \frac{2}{3}\pi\right)\right)\right)^{-1} , \\ r_+^2 &= \left(\frac{m}{3q^2}\left(1 + 2\cos\left(\frac{\theta}{3} + \frac{4}{3}\pi\right)\right)\right)^{-1} , \\ r_-^2 &= \left(\frac{m}{3q^2}\left(1 + 2\cos\left(\frac{\theta}{3}\right)\right)\right)^{-1} , \end{aligned} \tag{2.9}$$

with

$$\theta = \arctan\left(\frac{3\sqrt{3}q^2\sqrt{4m^3l^2 - 27q^4}}{2m^3l^2 - 27q^4}\right) , \tag{2.10}$$

where r_+ and r_- represent locations of the outer and inner horizons, respectively.

Since the gauge potential $\mathcal{A}_t$ must vanish at the outer horizon, the charge Q and the chemical potential μ are related as $Q = r_+^2\mu$. This background is induced by the back reaction of the $U(1)$ charge as in [36]. In the case of the bulk-filling D7-brane, the $U(1)$ charge is not identified as the $U(1)$ baryon charge [22]. Even though, there are no baryons in the field theory dual of the STU solution, we can expect that the baryon charge has similar property to the R-charges in the STU solution. In accordance with this analogy, μ which is the asymptotic value of $\mathcal{A}_t$ can be interpreted as the chemical potential for the $U(1)$ baryon charge. Hence, we can interpret our results as a calculation for the QGP at finite temperature in the presence of the $U(1)$ baryon charge.

Now there are two theories, which are obtained from the bulk-filling D7-brane and the STU model as mentioned above. The different point is the presence of the Chern-Simons term in the later case. The aim of this paper is to complete the linear order hydrodynamics in the equations of motion for RN-AdS$_5$ with the Chern-Simons term. The study without the Chern-Simons term has been done [19, 20].

For the well-defined variational principle in spacetime with boundary, we need the Gibbons-Hawking term defined as

$$S_{\rm GH} = \frac{1}{8\pi G_5}\int \mathrm{d}^4x\sqrt{-g^{(4)}}K \ , \tag{2.11}$$

where $g^{(4)}_{\mu\nu}$ and K represent the four-dimensional induced metric and the extrinsic curvature on the boundary, respectively. The integration is taken on the boundary.

The Hawking temperature of RN-AdS$_5$ background is given as

$$T = \frac{r_+^2 f'(r_+)}{4\pi l^2} = \frac{r_+}{\pi l^2}\Big(1 - \frac{1}{2}\frac{q^2 l^2}{r_+^6}\Big) \equiv \frac{1}{2\pi b}\Big(1 - \frac{a}{2}\Big) \ , \tag{2.12}$$

where a and b are defined as

$$a \equiv \frac{q^2 l^2}{r_+^6} \qquad \text{and} \qquad b \equiv \frac{l^2}{2r_+} \ . \tag{2.13}$$

We can see that the value of a can be taken in $0 \leq a < 2$. These two parameters can be rewritten in terms of the chemical potential and temperature as

$$a = 2 - \frac{4}{1 + \sqrt{1 + 4(\tilde{\mu}/T)^2}} \qquad \text{and} \qquad b = \frac{1}{\pi T(1 + \sqrt{1 + 4(\tilde{\mu}/T)^2}\ )} \ , \tag{2.14}$$

where $\tilde{\mu}$ is a rescaled chemical potential as $\tilde{\mu} \equiv \mu\sqrt{16\pi G_5/(3(\pi e l)^2)}$ [*].

3 Hydrodynamics of RN-AdS$_5$ background

Let us now consider classical perturbations on the RN-AdS$_5$ background as

$$g_{mn} \equiv g^{(0)}_{mn} + h_{mn} \quad \text{and} \quad \mathcal{A}_m \equiv A^{(0)}_m + A_m \ , \tag{3.1}$$

where ($g^{(0)}_{mn}$ and $A^{(0)}_m$) and (h_{mn} and A_m) denote the RN-AdS$_5$ background (2.7) and the perturbations, respectively. We choose the following gauge conditions,

$$h_{rm} = 0 \quad \text{and} \quad A_r = 0 \ , \tag{3.2}$$

and use Fourier expansion in which the momentum lies on the z-direction,

$$\begin{aligned} h_{\mu\nu}(t,z,r) \ &= \int \frac{\mathrm{d}^2 k}{(2\pi)^2}\ \mathrm{e}^{-i\omega t + ikz} h_{\mu\nu}(\omega,k,r) \ , \\ A_\mu(t,z,r) \ &= \int \frac{\mathrm{d}^2 k}{(2\pi)^2}\ \mathrm{e}^{-i\omega t + ikz} A_\mu(\omega,k,r) \ , \end{aligned} \tag{3.3}$$

where μ and ν run through the four-dimensional spacetime except for the radial direction. The perturbations can be categorized to the following three types,

[*]For the R-charge, $\tilde{\mu} \equiv \mu/(2\sqrt{3}\pi)$, while for the brane charge, $\tilde{\mu} \equiv \mu\sqrt{N_f/(3N_c\pi^2)}$.

- vector type : $h_{xt},\ h_{yt},\ h_{xz},\ h_{yz}$ and $A_x,\ A_y$
- scalar type : $h_{tt},\ h_{tz},\ h_{xx}=h_{yy},\ h_{zz}$ and $A_t,\ A_z$
- tensor type : $h_{xy},\ h_{xx}=-h_{yy}$

Taking linear order for the perturbations in the equations of motion, it turns out that the Chern-Simons term contributes to only the vector type. Thus in this study, we consider the vector type.

3.1 Decoupling of the equations of motion

We start with introducing a dimensionless coordinate u which is normalized by the outer horizon as

$$u \equiv \frac{r_+^2}{r^2}\ . \tag{3.4}$$

In this notation, the horizon and the boundary are located at $u=1$ and 0, respectively.

Let us see the equations of motion (2.5a) and (2.5b) for the perturbation fields. These are given as

$$0 = h_t^{x\prime\prime} - \frac{1}{u}h_t^{x\prime} - \frac{b^2}{uf(u)}(\omega k h_z^x + k^2 h_t^x) - 3auB_x'\ , \tag{3.5a}$$

$$0 = h_t^{y\prime\prime} - \frac{1}{u}h_t^{y\prime} - \frac{b^2}{uf(u)}(\omega k h_z^y + k^2 h_t^y) - 3auB_y'\ , \tag{3.5b}$$

$$0 = kf(u)h_z^{x\prime} + \omega h_t^{x\prime} - 3a\omega uB_x\ , \tag{3.5c}$$

$$0 = kf(u)h_z^{y\prime} + \omega h_t^{y\prime} - 3a\omega uB_y\ , \tag{3.5d}$$

$$0 = h_z^{x\prime\prime} + \frac{(u^{-1}f(u))'}{u^{-1}f(u)}h_z^{x\prime} + \frac{b^2}{uf^2(u)}(\omega^2 h_z^x + \omega k h_t^x)\ , \tag{3.5e}$$

$$0 = h_z^{y\prime\prime} + \frac{(u^{-1}f(u))'}{u^{-1}f(u)}h_z^{y\prime} + \frac{b^2}{uf^2(u)}(\omega^2 h_z^y + \omega k h_t^y)\ , \tag{3.5f}$$

and

$$0 = B_x'' + \frac{f'(u)}{f(u)}B_x' + \frac{b^2}{uf^2(u)}(\omega^2 - k^2 f(u))B_x - \frac{1}{f(u)}h_t^{x\prime} - \tilde{\kappa}\frac{ik}{f(u)}B_y\ , \tag{3.5g}$$

$$0 = B_y'' + \frac{f'(u)}{f(u)}B_y' + \frac{b^2}{uf^2(u)}(\omega^2 - k^2 f(u))B_y - \frac{1}{f(u)}h_t^{y\prime} + \tilde{\kappa}\frac{ik}{f(u)}B_x\ , \tag{3.5h}$$

with

$$f(u) = (1-u)(1+u-au^2)\ , \quad \tilde{\kappa} \equiv \frac{64e^2Qb^4}{l^5}\kappa\ , \quad B_{x(y)} \equiv \frac{A_{x(y)}}{\mu} = \frac{l^4}{4Qb^2}A_{x(y)}\ , \tag{3.6}$$

where the prime implies the derivative with respect to u. As we explain below, there are four independent variables. (3.5e) can be derived from (3.5a) and (3.5c). $h_z^{x\prime}(u)$ could be expressed in terms of $h_t^{x\prime}(u)$ and $B_x(u)$ through (3.5c). From (3.5a) and (3.5c) we can obtain a second order differential equation for $h_t^{x\prime}(u)$ with $B_x(u)$. The same structure is hold for (3.5b), (3.5d) and (3.5f). Together with (3.5g) and (3.5h), we treat $h_t^{x\prime}(u)$, $B_x(u)$, $h_t^{y\prime}(u)$ and $B_y(u)$ as the four independent variables.

In order to solve the coupled equations of motion, it might be convenient to introduce master variables [37]. We first take the following combinations [19]:

$$\begin{aligned} \Theta_{x\pm} &\equiv \frac{1}{u}h_t^{x\prime} - \Big(3a - \frac{C_\pm}{u}\Big)B_x \ , \\ \Theta_{y\pm} &\equiv \frac{1}{u}h_t^{y\prime} - \Big(3a - \frac{C_\pm}{u}\Big)B_y \ , \end{aligned} \tag{3.7}$$

with

$$C_\pm \equiv 1 + a \pm \sqrt{(1+a)^2 + 3ab^2k^2} \ .$$

Using these variables, we can obtain the following equations:

$$\begin{aligned} 0 &= \Theta''_{x\pm} + \frac{(u^2f(u))'}{u^2f(u)}\Theta'_{x\pm} + \frac{b^2(\omega^2 - k^2f(u)) - uf(u)C_\pm}{uf^2(u)}\Theta_{x\pm} - \frac{iC_\pm k\tilde{\kappa}}{C_0f(u)}(\Theta_{y+} - \Theta_{y-}) \ , \\ 0 &= \Theta''_{y\pm} + \frac{(u^2f(u))'}{u^2f(u)}\Theta'_{y\pm} + \frac{b^2(\omega^2 - k^2f(u)) - uf(u)C_\pm}{uf^2(u)}\Theta_{y\pm} + \frac{iC_\pm k\tilde{\kappa}}{C_0f(u)}(\Theta_{x+} - \Theta_{x-}) \ , \end{aligned} \tag{3.8}$$

where $C_0 \equiv C_+ - C_-$. Comparing with the previous work [19], there still remain non-diagonal terms in the r.h.s of the equations (3.8). In order to diagonalize these equations, we rewrite these by using a matrix form,

$$0 = \Theta'' + \frac{(u^2f(u))'}{u^2f(u)}\Theta' + \Omega(u)\Theta \ , \qquad \text{with} \qquad \Theta \equiv (\Theta_{x+}, \Theta_{x-}, \Theta_{y+}, \Theta_{y-})^{\mathrm{T}} \ , \tag{3.9}$$

and choose a matrix Λ which leads a diagonal matrix $\widetilde{\Omega} \equiv \Lambda^{-1}\Omega\Lambda$ as

$$\Lambda^{-1} = \frac{1}{2C_0}\begin{pmatrix} -i\frac{C_-\tilde{\kappa}}{D_+k^2} & -i\frac{C_0^2 - C_0D_+ - 2(1+a)\tilde{\kappa}k}{2D_+k^3} & -\frac{C_-\tilde{\kappa}}{D_+k^2} & -\frac{C_0^2 - C_0D_+ - 2(1+a)\tilde{\kappa}k}{2D_+k^3} \\ i\frac{C_-\tilde{\kappa}k}{D_+} & i\frac{C_0^2 + C_0D_+ - 2(1+a)\tilde{\kappa}k}{2D_+} & \frac{C_-\tilde{\kappa}k}{D_+} & \frac{C_0^2 + C_0D_+ - 2(1+a)\tilde{\kappa}k}{2D_+} \\ -i\frac{C_-\tilde{\kappa}}{D_-k^2} & i\frac{C_0^2 - C_0D_- + 2(1+a)\tilde{\kappa}k}{2D_-k^3} & \frac{C_-\tilde{\kappa}}{D_-k^2} & -\frac{C_0^2 - C_0D_- + 2(1+a)\tilde{\kappa}k}{2D_-k^3} \\ i\frac{C_-\tilde{\kappa}k}{D_-} & -i\frac{C_0^2 + C_0D_- + 2(1+a)\tilde{\kappa}k}{2D_-} & -\frac{C_-\tilde{\kappa}k}{D_-} & \frac{C_0^2 + C_0D_- + 2(1+a)\tilde{\kappa}k}{2D_-} \end{pmatrix} , \tag{3.10}$$

with

$$D_\pm \equiv \sqrt{(C_0 \pm \tilde{\kappa}k)^2 \pm 4\tilde{\kappa}kC_-} = 2(1+a) \pm \tilde{\kappa}k + \frac{3ab^2}{1+a}k^2 + \mathcal{O}(k^3)\ .$$

It might be important that the matrix Λ is independent of u. Eigenvectors $\widetilde{\Theta} \equiv \Lambda^{-1}\Theta \equiv (\widetilde{\Theta}_{1-}, \widetilde{\Theta}_{2-}, \widetilde{\Theta}_{1+}, \widetilde{\Theta}_{2+})^{\mathrm{T}}$ are not mixed by the derivatives with respect to u. Moreover, defined new variables $\widetilde{\Theta}$ correspond to helicity bases on the xy-plane.

The diagonalized equations are then given as

$$0 = \widetilde{\Theta}'' + \frac{(u^2 f(u))'}{u^2 f(u)}\widetilde{\Theta}' + \widetilde{\Omega}(u)\widetilde{\Theta}\ , \tag{3.11}$$

with the diagonal matrix

$$\begin{aligned}\widetilde{\Omega} = &\frac{b^2}{uf^2(u)}(\omega^2 - f(u)k^2) - \frac{1+a}{f(u)} \\ &+\frac{1}{2f(u)}\mathrm{diag}\Big(- D_- + \tilde{\kappa}k,\ D_- + \tilde{\kappa}k,\ -D_+ - \tilde{\kappa}k,\ D_+ - \tilde{\kappa}k\Big).\end{aligned} \tag{3.12}$$

This $\widetilde{\Theta}$ is a master variable with which we shall work.

3.2 Solutions under the hydrodynamic approximation

The equation of motion (3.11) is an ordinary second order differential equation with a regular singular point at $u = 1$. Thus, the solution can be generally written as

$$\widetilde{\Theta}_i = (\widetilde{C}_1)_i(1-u)^{-\nu}(\widetilde{\Phi}_1)_i + (\widetilde{C}_2)_i(1-u)^{\nu}(\widetilde{\Phi}_2)_i \quad \text{with} \quad \nu = i\frac{\omega}{4\pi T}\ , \tag{3.13}$$

where $\widetilde{C}_1$ and $\widetilde{C}_2$ are integral constants, and $\widetilde{\Phi}_1(u)$ and $\widetilde{\Phi}_2(u)$ are regular functions at $u = 1$. Index i stands for the i-th component of vectors, which takes $(1-)$, $(2-)$, $(1+)$ and $(2+)$. The first term and the second term in the r.h.s. represent a wave toward the horizon (in-going solution) and a wave away from the horizon (out-going solution), respectively*. We choose the in-going solution ($\widetilde{C}_2 = 0$) as the boundary condition at the horizon. The remaining constant $\widetilde{C}_1$ will be fixed later.

We consider the expansion in which k and ω are small comparing with the temperature. In this approximation, the perturbation variables describe the hydrodynamics as the large distance and long time-scale classical wave on the equilibrium state. In the vector mode, order of k^2 is the same as ω. In the present case, due to the Chern-Simons term, the

*For a clear explanation, see ref. [38].

expansion stars from k, which we call the first order. The second and the third orders are given by (ω, k^2) and $(k\omega, k^3)$, respectively. The expansion of $\widetilde{\Phi}_1(u)$ might be

$$(\widetilde{\Phi}_1)_i = \widetilde{\rho}_i(\widetilde{X})_i = \widetilde{\rho}_i \left[(\widetilde{X}_0)_i + k(\widetilde{X}_k)_i + \omega(\widetilde{X}_\omega)_i + k^2(\widetilde{X}_{k^2})_i + \mathcal{O}(k\omega, k^3)\right] \ , \tag{3.14}$$

where we have introduced $\widetilde{\rho}_i(u)$ for later convenience. Then, the equation of motion (3.11) for $\widetilde{X}$ can be written in the following form,

$$0 = \Big(u^2 f(u)\widetilde{\rho}_i^{\,2}(u)\Big((1-u)^{-\nu}(\widetilde{X})_i\Big)'\Big)' + u^2 f(u)\widetilde{\rho}_i(u)\ V_i(u)\ (1-u)^{-\nu}(\widetilde{X})_i \ , \tag{3.15}$$

with

$$V_i \equiv \widetilde{\rho}_i\,'' + \frac{(u^2 f(u))'}{u^2 f(u)}\widetilde{\rho}_i\,' + \widetilde{\Omega}_i(u)\widetilde{\rho}_i \ .$$

We here take $\widetilde{\rho}_i(u)$ so that V starts from the first order in the expansion of k and ω,

$$\widetilde{\rho} \equiv \Big(-\frac{2(1+a)}{u} + 3a,\ 1,\ -\frac{2(1+a)}{u} + 3a,\ 1\Big)^{\mathrm{T}} \ .$$

Then we could solve the differential equation order by order. The equation of each order takes the form of $(F(\widetilde{X}_*)_i')' = G$ schematically. Here, F is a function of u, and G contains only lower orders of $\widetilde{X}_*$. Therefore we can obtain the solution by integrating these equations twice.

Two integration constants in this procedure are related to the two integration constants in (3.13). We have imposed an in-going condition at the horizon and the singularity has been already factorized. Then, one of the integration constants in $\widetilde{X}_*(u)$ must be taken such that they are regular. The other integration constant can be absorbed into the normalization constant $\widetilde{C}_1$. In order to fix the constant $\widetilde{C}_1$, we use the following relation given by (3.5a), (3.5b) and (3.7),

$$\begin{aligned} u^2\Theta'_{x\pm} - uC_\pm B'_x\Big|_{u=0} &= b^2\Big(\omega k(h_z^x)^0 + k^2(h_t^x)^0\Big) - C_\pm(B_x)^0 \ , \\ u^2\Theta'_{y\pm} - uC_\pm B'_y\Big|_{u=0} &= b^2\Big(\omega k(h_z^y)^0 + k^2(h_t^y)^0\Big) - C_\pm(B_y)^0 \ , \end{aligned} \tag{3.16}$$

where $(h_t^{x(y)})^0$, $(h_z^{x(y)})^0$ and $(B_{x(y)})^0$ mean constant values at the boundary. We rewrite the l.h.s. of these equations in terms of $\widetilde{\Phi}_1(u)$ and substitute the solution of the differential equations. Solving the equations (3.16), we can then obtain the expression of $\widetilde{C}_1$ in terms of the boundary values of perturbations variables.

Following the procedure above, we can obtain the solutions around the boundary up to $\mathcal{O}(\omega, k^2)$:

$$
\begin{aligned}
h_t^x(u) &= (h_t^x)^0 - b^2\Big(k^2(h_t^x)^0 + \omega k(h_z^x)^0\Big)u \\
&\quad + \frac{1}{8P}\Big\{4b\Big(k^2(h_t^x)^0 + \omega k(h_z^x)^0\Big) + 3ia\Big(4\omega - \frac{3ab^2}{(1+a)^2}\omega k^2\Big)(B_x)^0\Big\}u^2 \ , \qquad (3.17a) \\
h_t^y(u) &= (h_t^y)^0 - b^2\Big(k^2(h_t^y)^0 + \omega k(h_z^y)^0\Big)u \\
&\quad + \frac{1}{8P}\Big\{4b\Big(k^2(h_t^y)^0 + \omega k(h_z^y)^0\Big) + 3ia\Big(4\omega - \frac{3ab^2}{(1+a)^2}\omega k^2\Big)(B_y)^0\Big\}u^2 \ , \qquad (3.17b) \\
h_z^x(u) &= (h_z^x)^0 + b^2\Big(\omega^2(h_z^x)^0 + \omega k(h_t^x)^0\Big)u \\
&\quad - \frac{1}{4P}\Big\{2b\Big(\omega^2(h_z^x)^0 + \omega k(h_t^x)^0\Big) + \frac{3ab}{1+a}\omega k(B_x)^0\Big\}u^2 \ , \qquad (3.17c) \\
h_z^y(u) &= (h_z^y)^0 + b^2\Big(\omega^2(h_z^y)^0 + \omega k(h_t^y)^0\Big)u \\
&\quad - \frac{1}{4P}\Big\{2b\Big(\omega^2(h_z^y)^0 + \omega k(h_t^y)^0\Big) + \frac{3ab}{1+a}\omega k(B_y)^0\Big\}u^2 \ , \qquad (3.17d) \\
B_x(u) &= (B_x)^0 + b^2k^2(B_x)^0 u\log u \\
&\quad + \frac{i}{8(1+a)^2P}\Big\{\frac{1}{1+a}\Big(-12a(1+a)^2\omega + 2i(1+a)(2-a)b\omega^2 \\
&\qquad\qquad -(12+8a-17a^2-4a^3)b^2\omega k^2\Big)(B_x)^0 \\
&\qquad + \frac{\kappa}{2(1+a)}\Big(2(1+a)(4+a)bk^3 - 4i(4+2a+a^2)\omega k\Big)(B_y)^0 \\
&\qquad + \Big(4i(1+a)bk^2 + 6ab^2\omega k^2\Big)(h_t^x)^0 + 4i(1+a)b\omega k(h_z^x)^0 \\
&\qquad + 2\kappa b\Big(k^3(h_t^y)^0 + \omega k^2(h_z^y)^0\Big)\Big\}u \ , \qquad (3.17e) \\
B_y(u) &= (B_y)^0 + b^2k^2(B_y)^0 u\log u \\
&\quad + \frac{i}{8(1+a)^2P}\Big\{\frac{1}{1+a}\Big(-12a(1+a)^2\omega + 2i(1+a)(2-a)b\omega^2 \\
&\qquad\qquad -(12+8a-17a^2-4a^3)b^2\omega k^2\Big)(B_y)^0 \\
&\qquad - \frac{\kappa}{2(1+a)}\Big(2(1+a)(4+a)bk^3 - 4i(4+2a+a^2)\omega k\Big)(B_x)^0 \\
&\qquad + \Big(4i(1+a)bk^2 + 6ab^2\omega k^2\Big)(h_t^y)^0 + 4i(1+a)b\omega k(h_z^y)^0 \\
&\qquad - 2\kappa b\Big(k^3(h_t^x)^0 + \omega k^2(h_z^x)^0\Big)\Big\}u \ , \qquad (3.17f)
\end{aligned}
$$

where P gives a pole structure with a diffusion constant D,

$$P = i\omega - Dk^2 = i\omega - \frac{b}{2(1+a)}k^2. \tag{3.18}$$

We can observe the effect of Chern-Simons term in $B_{x(y)}(u)$ via the parameter κ.

It should be worth to mention the different structure in the equation of motion (3.11), which may be crucial to obtain analytical solutions. The first and the third components of (3.11) can be solved analytically up to the second order $\mathcal{O}(\omega, k^2)$, while the second and the fourth components can be solved up to the third order $\mathcal{O}(k\omega, k^3)$. Since the second and the fourth components provide the pole structure, the equation (3.18) would be modified to

$$\mathcal{P} = i\omega - Dk^2 \pm \frac{ab}{4(1+a)^2}\tilde{\kappa}k^3 \ , \tag{3.19}$$

where $\pm$ correspond to the helicity. We can observe that the Chern-Simons term now affects to the pole structure. This was firstly calculated in [39]. Our result is consistent with theirs.

4 The dual quark-gluon-plasma

We have been studying the dual gravity side so far and obtained the solutions. In this section, we move to the field theory side and consider the effect of the Chern-Simons term to the retarded two-point Green function in the dual QGP at finite temperature with the $U(1)$ baryon density.

4.1 Boundary action

In order to obtain the retarded two-point Green function following the GKP-W relation [2, 3], we need the bilinear-part of the regularized boundary action at $u=0$. Together with the Gibbons-Hawking term (2.11), we need the following counter term for the regularized action at the boundary,

$$S_{\text{ct}} = S_{\text{ct, gravity}} + S_{\text{ct, gauge}} \ , \tag{4.1}$$

where

$$S_{\text{ct, gravity}} = \frac{1}{8\pi G_5}\int \mathrm{d}^4x\sqrt{-g^{(4)}}\Big(\frac{3}{l} - \frac{l}{4}R^{(4)}\Big) \ , \tag{4.2a}$$

$$S_{\text{ct, gauge}} = \frac{l}{8e^2}\log u \int \mathrm{d}^4x\sqrt{-g^{(4)}} \ \mathcal{F}_{mn}\mathcal{F}^{mn} \ , \tag{4.2b}$$

where $R^{(4)}$ is the curvature on the boundary. $S_{\text{ct, gravity}}$ is given in [40]. On the other hand, $S_{\text{ct, gauge}}$ is obtained to cancel the logarithmic divergence coming from gauge field fluctuations.

The boundary action derived from (2.1) is

$$S^{(0)} = \frac{l^3}{256\pi b^4 G_5 u^2}\int \frac{\mathrm{d}^2k}{(2\pi)^2}\bigg\{\frac{uf'(u)}{f(u)}(h_t^x)^2 + h_t^x\Big(h_t^x - 3uh_t^{x\prime}\Big) - f(u)h_z^x\Big(h_z^x - 3uh_z^{x\prime}\Big) \\ +3aB_x\Big(h_t^x - f(u)B_x{}'\Big)\bigg\} \; . \tag{4.3}$$

The Gibbons-Hawking term (2.11) and the counter term (4.1) are

$$S_{\text{GH}}^{(0)} = \frac{l^3}{256\pi b^4 G_5 u^2}\int \frac{\mathrm{d}^2k}{(2\pi)^2}\bigg\{-\frac{uf'(u)}{f(u)}(h_t^x)^2 - 4(h_t^x)^2 - uf'(u)(h_z^x)^2 + 4uh_t^x h_t^{x\prime} \\ +4f(u)\Big((h_z^x)^2 - uh_z^x(h_z^x)'\Big)\bigg\} \; , \tag{4.4}$$

$$S_{\text{ct}}^{(0)} = \frac{3l^3}{256\pi b^4 G_5 u^2\sqrt{f(u)}}\int \frac{\mathrm{d}^2k}{(2\pi)^2}\bigg\{(h_t^x)^2 - f(u)(h_z^x)^2 \\ +ab^2k^2u^2 f(u)\log u\Big(B_x^2 + B_y^2\Big)\bigg\} \; . \tag{4.5}$$

Then, the regularized boundary action is given as:

$$\begin{aligned} S_{\text{st}} &= \lim_{u\to 0}(S^{(0)} + S_{\text{GH}}^{(0)} + S_{\text{ct}}^{(0)}) \\ &= \lim_{u\to 0}\frac{l^3}{256\pi b^4 G_5}\int \frac{\mathrm{d}^2k}{(2\pi)^2}\bigg\{\frac{1}{u}\Big(h_t^x(-k)h_t^{x\prime}(k) - h_z^x(-k)h_z^{x\prime}(k)\Big) \\ &\qquad + \frac{1+a}{2}\Big(3h_t^x(-k)h_t^x(k) + ah_z^x(-k)h_z^x(k)\Big) \\ &\qquad - 3aB_x(-k)\Big(h_t^x(k) + B_x{}'(k)\Big) \\ &\qquad + 3ab^2k^2\log u\Big(B_x(-k)B_x(k) + B_y(-k)B_y(k)\Big)\bigg\} \; . \end{aligned} \tag{4.6}$$

We can see that the Chern-Simons term does not explicitly enter the boundary action. This can be expected from the original property of the Chern-Simons term.

4.2 Retarded two-point Green functions

Substituting the solutions (3.17a)-(3.17f) to the regularized boundary action (4.6), we can obtain the retarded two-point Green functions up to $\mathcal{O}(\omega, k^2)$ as

$$G_{xt\ xt}(k,\omega) = G_{yt\ yt}(k,\omega) = \frac{l^3}{128\pi b^3 G_5}\frac{k^2}{P} \; , \tag{4.7a}$$

$$G_{xt\ xz}(k,\omega) = G_{yt\ yz}(k,\omega) = -\frac{l^3}{128\pi b^3 G_5}\frac{\omega k}{P} \ , \tag{4.7b}$$

$$G_{xz\ xz}(k,\omega) = G_{yz\ yz}(k,\omega) = \frac{l^3}{128\pi b^3 G_5}\frac{\omega^2}{P} \ , \tag{4.7c}$$

$$G_{xt\ x}(k,\omega) = G_{yt\ y}(k,\omega) = -\frac{Q}{4e^2 l^3(1+a)}\left(\frac{2bk^2 + 4i(1+a)\omega}{P}\right) \ , \tag{4.7d}$$

$$G_{xz\ x}(k,\omega) = G_{yz\ y}(k,\omega) = \frac{Qb}{e^2 l^3(1+a)}\frac{\omega k}{P} \ , \tag{4.7e}$$

$$G_{x\ yt}(k,\omega) = -G_{y\ xt}(k,\omega) = -\frac{768Q^2 b^5}{(1+a)l^8}\frac{k^3}{P}\kappa \ , \tag{4.7f}$$

$$G_{x\ x}(k,\omega) = G_{y\ y}(k,\omega) = \frac{3ial}{4e^2(1+a)b^2}\frac{\omega}{P} \ , \tag{4.7g}$$

$$G_{x\ y}(k,\omega) = -G_{y\ x}(k,\omega) = \frac{6Qb^2}{(1+a)^2 l^4}\left(\frac{i(4+a)bk^3 + 2(4+2a+a^2)\omega k}{P}\right)\kappa \ , \tag{4.7h}$$

$$(\text{others}) = 0 \ , \tag{4.7i}$$

where P is given by (3.18), which gives the pole structure. In the above expressions, we restored the gauge field $A_{x(y)}$ and κ by using the relations (3.6) and rise and lower the indices by using the boundary Minkowski metric. It can be seen that the Chern-Simons term affects $G_{x\ yt}(k,\omega)$ and $G_{x\ y}(k,\omega)$ via κ.

The presence of non-vanishing $G_{x\ y}(k,\omega)$ indicates the existence of chiral magnetic conductivity introduced in [41], which is defined by $J_i = \sigma_B \mathcal{B}_i$ where $\mathcal{B}_i$ is the magnetic field. Using $\mathcal{B}_x(k) = (-ik)A_y(k)$, σ_B in hydrodynamic limit can be read off:

$$\begin{aligned} \sigma_B(\omega,k) &= \frac{1}{-ik}G_{x\ y}(k,\omega) \\ &= -\frac{6Qb^2}{(1+a)^2 l^4 P}\Big(-(4+a)bk^2 + 2i(4+2a+a^2)\omega\Big)\kappa \ . \end{aligned} \tag{4.8}$$

Taking the limit $\omega \to 0$ first and then $k \to 0$, the DC conductivity σ_B^0 becomes

$$\sigma_B^0 = \lim_{\omega\to 0}\sigma_B(\omega,k) = -\frac{12Q(4+a)b^2}{l^4(1+a)}\kappa \ . \tag{4.9}$$

The presence of magnetic conductivity is the most important consequence of the Chern-Simons term. The physical origin is due to the effect of the $U(1)_R$ anomaly. In [41] the magnetic conductivity due to the axial anomaly was discussed: The magnetic field can change the spin direction and the momentum is tied with the spin, making left and right handed particles move in opposite direction. Therefore the magnetic field can induce an electric current proportional to the difference of left and right handed zero modes, which

is nothing but the chiral anomaly. In our case the role of chiral symmetry is replaced by the $U(1)_R$ symmetry.

The magnetic conductivity in holographic set-up was calculated in [33, 34]. Our result agrees with [33] * but not with [34]. The difference can be traced to that of the set-ups†.

It is interesting to express the magnetic conductivity in terms of the boundary variables, i.e. the temperature and the chemical potential through the equations (2.14):

$$\sigma_B^0 = -6\kappa\mu(\widetilde{\sigma}_B)_0, \qquad (\widetilde{\sigma}_B)_0 = \frac{3\gamma+1}{3\gamma-1}, \tag{4.10}$$

where $\gamma = \sqrt{1+4(\tilde{\mu}/T)^2}$. The behaviors in low and high temperature regions are given by

$$(\widetilde{\sigma}_B)_0 = 1 + \frac{1}{3}\frac{T}{\tilde{\mu}} + \frac{1}{18}\left(\frac{T}{\tilde{\mu}}\right)^2 + \mathcal{O}\left((T/\tilde{\mu})^3\right) \quad : \text{low } T\ , \tag{4.11a}$$

$$(\widetilde{\sigma}_B)_0 = 2 - 3\left(\frac{\tilde{\mu}}{T}\right)^2 + \mathcal{O}\left((\tilde{\mu}/T)^6\right) \quad : \text{high } T\ . \tag{4.11b}$$

It should be noticed that at low temperature, the magnetic conductivity increases linearly in T starting from a non-vanishing value and saturate to a finite value. Interestingly, its value at the large temperature is just twice of the value at zero temperature. Such behavior should be contrasted with that of electric conductivity [20]

$$\sigma_E = \frac{le_E^2}{8e^2}\frac{(2-a)^2}{b(1+a)^2} \equiv \frac{le_E^2}{8e^2}\cdot 8\pi\tilde{\mu}(\widetilde{\sigma}_E)_0, \tag{4.12}$$

which is $\sim T^2$ and $\sim T$ in low and high temperature regime respectively, as one can see from

$$(\widetilde{\sigma}_E)_0 = \frac{1}{9}\left(\frac{T}{\tilde{\mu}}\right)^2 + \frac{5}{54}\left(\frac{T}{\tilde{\mu}}\right)^3 + \mathcal{O}\left((T/\tilde{\mu})^5\right) \quad : \text{low } T\ , \tag{4.13a}$$

$$(\widetilde{\sigma}_E)_0 = \frac{T}{\tilde{\mu}} - 5\frac{\tilde{\mu}}{T} + 26\left(\frac{\tilde{\mu}}{T}\right)^3 + \mathcal{O}\left((\tilde{\mu}/T)^5\right) \quad : \text{high } T\ . \tag{4.13b}$$

Whole behavior of magnetic and electric conductivities are drawn as functions of the $T/\tilde{\mu}$ in Figure 1.

---

* In terms of the variables in [33], our magnetic conductivity (4.9) should be compared with ξ_B and two are related by $\sigma_B^0 = \frac{3}{2}\xi_B$, where $\xi_B = -\frac{\sqrt{3}(3R^4+m)q\kappa}{4\pi G_5 mR^2}$, with R being the horizon radius.

† In [34], the author introduced two gauge fields which are associated with the axial and vector $U(1)$. He considers non-zero background charge for the axial $U(1)$ gauge field and perturbations only for the gauge field of vector $U(1)$. Then, the perturbation of gravity is decoupled from that of the gauge field. On the other hand, in our setup, we considered the background charge and perturbations for the same gauge field, and then, the gravitational perturbation couples to the gauge field perturbation. As a consequence, the DC magnetic conductivity of [34] is independent of temperature while our result depends on it.

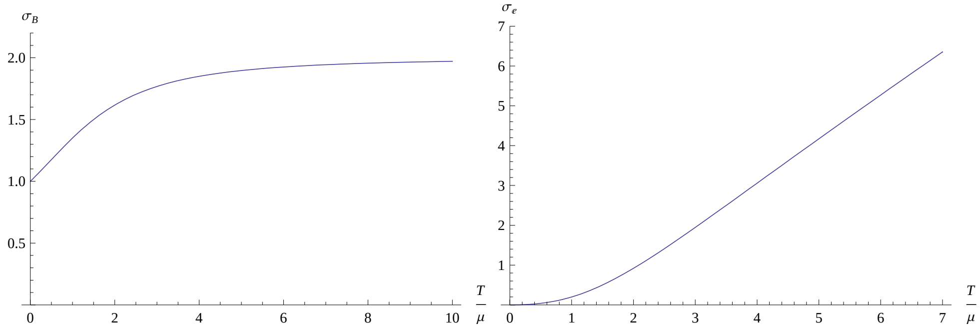

Figure 1: conductivities as functions of $T/\tilde{\mu}$. Left figure is $(\widetilde{\sigma}_B)_0$ and right one is $(\widetilde{\sigma}_E)_0$.

5 Summary

The RN-AdS$_5$ background may be considered as is the dual gravity for the QGP with finite temperature and the baryon density. One can add the Chern Simon term to the action. Although it does not change the solution, it can affect the fluctuation around the RN black hole. We have studied its effect in the hydrodynamics of the QGP coming. Although this interpretation is interesting from the perspective of more realistic holographic QCD, the theory is better known to come from the 10 dimensional STU solution with diagonal U(1) charges. From this point of view, the Chern-Simons term has a predetermined coupling.

We worked out the decoupling problem of equations of motion by using the master variables in the hydrodynamic analysis on the dual gravity. As a result, one of the effects of the Chern-Simons term appears in the retarded two-point Green function of $U(1)$ currents $G_{y\ x}(k,\omega)$ and $G_{x\ y}(k,\omega)$ in the dual QCD, which lead to the magnetic conductivity. This quantity is written in terms of the temperature and the chemical potential and compared with electric conductivity. The Chern-Simons term also contributes to the hydrodynamic pole structure in $\mathcal{O}(k^3)$.

In the case of the D7-brane effective theory, the Chern-Simons term in the five-dimensional gauge theory appears if we introduce the 3-form RR-flux which lies on the S^3 part of the world volume. The 3-form flux is the magnetic flux associated with the D5-brane, and baryons can be constructed by using the D5-brane [42]. It would be interesting to consider the relation between the effect of the Chern-Simons term and the magnetic flux of the

D5-brane.

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Appendix A Retarded Green function in AdS/CFT

In this appendix, we briefly summarize the retarded Green function of the boundary theory in AdS/CFT correspondence formulated by Son and Starinets [14]. To begin with, we consider perturbations on five-dimensional background. We refer x^μ and u as the four-dimensional and the radial coordinates, respectively, The boundary and the horizon locate at $u = 0$ and $u = 1$, respectively. We now suppose a solution of the equation of motion can be written as

$$\phi(u,x) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\ \mathrm{e}^{ikx} f_k(u)\phi(k) \tag{A.1}$$

A quadratic part of the surface action can be written as

$$S[\phi] = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \phi(-k)G(k,u)\phi(k)\Big|_{u=0}^{u=1}\ , \tag{A.2}$$

where $G(k,u)$ is a function of $f_{\pm k}(u)$ and $\partial_u f_{\pm k}(u)$. Applying GKP-W relation [2, 3] to spacetimes in the real-time, the formula for the two-point retarded Green functions has been defined as

$$G^{\mathrm{R}}(k) \equiv 2G(k,u)\Big|_{u=0}\ . \tag{A.3}$$

Lastly, we define the retarded Green functions we discuss in this paper:

$$\begin{aligned}
G_{\mu\nu\ \rho\sigma}(k,\omega) &\equiv -i\int \frac{\mathrm{d}^2 x}{(2\pi)^2}\ \mathrm{e}^{-i\omega t+ikz}\theta(t)\ \Big\langle [T_{\mu\nu}(t,z),\ T_{\rho\sigma}(0,0)] \Big\rangle\ ,\\
G_{\mu\nu\ \rho}(k,\omega) &\equiv -i\int \frac{\mathrm{d}^2 x}{(2\pi)^2}\ \mathrm{e}^{-i\omega t+ikz}\theta(t)\ \Big\langle [T_{\mu\nu}(t,z),\ J_{\rho}(0,0)] \Big\rangle\ ,\\
G_{\mu\ \nu}(k,\omega) &\equiv -i\int \frac{\mathrm{d}^2 x}{(2\pi)^2}\ \mathrm{e}^{-i\omega t+ikz}\theta(t)\ \Big\langle [J_{\mu}(t,z),\ J_{\nu}(0,0)] \Big\rangle\ ,
\end{aligned} \tag{A.4}$$

where the momentum in this paper is taken to z-direction, and the operators $T_{\mu\nu}(t,z)$ and $J_\mu(t,z)$ are the energy-momentum tensor and the $U(1)$ baryon current, respectively.

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